

## SPURIOUS ECCENTRICITIES OF DISTORTED BINARY COMPONENTS

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### ABSTRACT

I discuss the effect of physical distortion on the velocities of close binary components and how we may use the resulting distortion of velocity curves to constrain some properties of binary systems, such as inclination and mass ratio. Precise new velocities for 5 Cet convincingly detect these distortions with their theoretically predicted phase dependence. We can even use such distortions of velocity curves to test Lucy's theory of convective gravity darkening. The observed distortions for TT Hya and 5 Cet require the contact components of those systems to be gravity darkened, probably somewhat more than predicted by Lucy's theory, but clearly not as much as expected for a radiative star. These results imply there is no credible evidence for eccentric orbits in binaries with contact components. I also present some speculative analyses of the observed properties of a binary encased in a nonrotating common envelope, if such an object could actually exist, and discuss how the limb darkening of some recently calculated model atmospheres for giant stars may bias my results for velocity-curve distortions, as well as other results from a wide range of analyses of binary stars.

*Subject headings:* binaries: spectroscopic — stars: oscillations — stars: individual (5 Cet, AX Mon, TT Hya)

*Online material:* machine-readable table

### 1. INTRODUCTION

Two recent papers suggest the time is ripe to look critically at the effect of stellar distortion on velocities of variable stars. In the first, Miller et al. (2007) found a slightly eccentric orbit for the Algol binary TT Hya, a system that has gone through mass exchange and, in so doing, surely circularized its orbit. In the second, Wood et al. (2004) used velocity variations calculated for a rotating prolate spheroid as a possible mechanism for explaining long-period velocity variations of AGB stars. The first paper assumes these distortions are irrelevant; the second, that they are crucial.

Solutions of the velocity curves of close binary systems often have small eccentricities thought to be spurious. Lucy & Sweeney (1971, 1973) argued that most of these were statistical flukes resulting from errors of measurement. However, as we shall see in § 3, there are circumstances in which such eccentricities result from interesting physical processes, and for which the resulting distortions may be used to extract useful information about the mass ratios and inclinations of these systems. Such distortions from proximity effects were actually predicted by Sterne (1941), explored by Wilson & Sofia (1976) as a way of determining mass ratios and inclinations of X-ray binaries, and discussed in passing by Wilson (1979), but otherwise they do seem to have been ignored as a way of extracting information about binaries.

Red giants often have unexplained secondary periods much longer than their radial fundamentals (e.g. Wood et al. 2004). They manifest uncomfortably large radial-velocity variations on those periods (Wood et al. 2004; Hinkle et al. 2002), which imply pulsational excursions of the order of 30% of the radius of the star. These variations remain one of the burning mysteries of stellar astronomy (Derekas et al. 2006). They seem to have the sort of color variations expected of radial pulsation, yet there is no known mechanism for giving such long periods.

I shall explore the implications of distortion on observed radial velocities, starting with an assessment of the rotating spheroids of Wood et al. in § 2, then exploring how a binary encased in a common envelope might appear to us in § 2.1, and finally dis-

cussing the much less speculative effects of distortion on radial velocities of close binary components in § 3.

### 2. ROTATION OF A PROLATE SPHEROID

Wood et al. (2004) used a prolate spheroid rotating about its short axis to model velocity variations of AGB stars, although they ultimately rejected that idea. Since such a figure will project symmetrically onto the sky about its rotation axis, there would be no net velocity variation for a uniformly illuminated disc. So these variations must occur solely from the effects of limb and gravity darkening over the distorted surface (P. Wood 2007, private communication; Sterne 1941).

Let us make some standard assumptions about the rotating spheroid. Give it axes  $a$  and  $b$  in its equatorial plane and, for simplicity,  $b$  toward its rotational pole. Assume the mass is concentrated to its center, so the local gravity is proportional to  $1/r^2$ , and let the gravity darkening be convective,  $F_{\text{bol}} \sim \nabla \Omega^g$ , with  $g = 0.32$  following Lucy (1967), where  $\Omega$  is the gravitational potential. Assume total darkening to the limb ( $x = 1.0$ ), a safe assumption for such cool stars. Finally, assume that the radial velocity of the centroid of light is the radial velocity measured for the star. Now, if we write a computer program for these somewhat questionable assumptions, we can calculate the light and velocity curves for such a star, if any such object were to exist.

Figure 1 shows some velocity curves for a prolate spheroid with  $a = 0.50$  and  $b = 0.25$ , a rather extreme case that illustrates the model pretty well. For this combination of parameters, we get light variation of about 1.9 mag at  $i = 90^\circ$ , somewhat larger than seen in the AGB stars. The solid and dashed curves have  $x = 1.0$  with  $g = 0.0$  and  $x = 0.0$  with  $g = 0.32$ , respectively, to show the separate effects of limb and gravity darkening, which seem to cancel each other out in this calculation. The dotted curve, for  $x = 1.0$  with  $g = 0.32$ , shows the combined effect. Both limb and gravity darkening obviously cause significant velocity variations mimicking an eccentric orbit. The curve for pure limb darkening in Figure 1 may be fit with an orbit in which

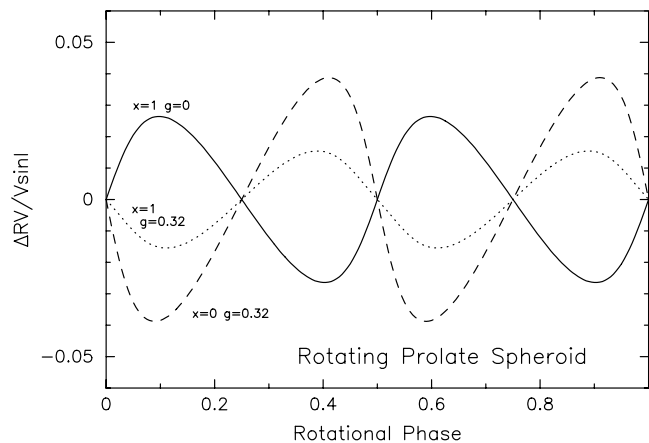


FIG. 1.—Velocity curves of a prolate spheroid with a 2:1 aspect ratio. The three curves show the effects of limb darkening alone (*solid*), of gravity darkening alone (*dashed*), and the combination of limb and gravity darkening (*dotted*). All assume  $i = 90^\circ$ .

$e = 0.21$  and  $\omega = 270^\circ$ . The case of pure gravity darkening gives  $e = 0.22$  and  $\omega = 87^\circ$ . Other aspect ratios ( $b/a$ ) give similar results but smaller amplitudes and somewhat smaller eccentricities for less elongation. More elongation gives very slightly bigger eccentricities.

These calculations for a rotating prolate spheroid do not represent the sort of long-period velocity variation observed in AGB stars. The eccentricities we find for this ellipsoidal model ( $e \sim 0.2$ ) are significantly smaller than the formal eccentricities of cool giants when compared to either the largest values measured or typical ones. Fitting the velocity shifts observed for nine stars with elliptical orbits gave  $e$  in the range 0.08–0.49 and  $\omega$  in the range  $195^\circ$ – $418^\circ$  (Hinkle et al. 2002; Wood et al. 2004), with medians  $e = 0.33$ ,  $\omega = 251^\circ$ , and  $K = 2.3 \text{ km s}^{-1}$ . The phase dependence of the calculated theoretical effect for gravity darkening is wrong in that it gives a rapid drop in velocity, as in a classical radial pulsation, not in the observed rapid rise. Furthermore, the velocity variation in these calculations ( $\Delta RV/v \sin i \sim 0.04$ ) is small enough to require a rotational velocity ( $58 \text{ km s}^{-1}$  for  $K = 2.3 \text{ km s}^{-1}$ ) prohibitively large for AGB stars, for which  $v_{\text{rot}} \lesssim 3 \text{ km s}^{-1}$  (Olivier & Wood 2003).

### 2.1. Speculations about an Encased Binary

It is not clear that any star, especially an AGB star, would be a prolate spheroid rotating about its fixed center of mass. The only way I can see to get a significant elongation is through tidal distortion in a close binary system, and the effects of that distortion on the velocities can be rather subtle, as we shall see in § 3. Wood et al. argue that some of these AGB stars may be coalesced binaries, presumably with double cores encased in a common envelope to account for the prolate shape, a radically imaginative idea deserving a closer look.

The size of such an object presents another fundamental problem with trying to use a rotating prolate spheroid to explain the light and radial-velocity variations of AGB stars. If the star has a double core, its period and linear scale are fixed by Kepler's laws. Consider, for example, a star with a double core having a period of 1588 days (4.35 yr), as discussed by Wood et al. for Z Eri. It would have a semimajor axis of 3 AU ( $640 R_\odot$ ) for two  $0.75 M_\odot$  components. Even in a binary consisting of two  $1.0 M_\odot$  components with a period of 750 days, the semimajor axis would be about  $440 R_\odot$ . The radii of these objects would be of the order of their semimajor axes. Wood et al. estimate radii near  $170 R_\odot$

for the stars with long secondary periods, so encased binaries with the right periods to explain the observed long-term velocity variations would seem to be too large. Also, if these stars were synchronously rotating, admittedly a completely unphysical condition, our two examples would have  $v_{\text{rot}}$  of 20 and  $30 \text{ km s}^{-1}$ , respectively, much larger than the  $v \sin i$ 's of AGB stars.

The idea of a binary totally encased in a common envelope seems preposterous, since the gravitational equipotential surfaces above the two cores, which presumably help control the density structure of the star, become very complicated with three Lagrangian points at which the gravitational acceleration vanishes (e.g., Kopal 1959, their Figs. 7-7 through 7-10). However, this may be no worse than the generally accepted idea of a contact binary containing a single such Lagrangian point. Given these circumstances, however, it is not clear to me that any star could be stable with a surface bigger than in a typical contact binary (one with its surface between the first and second Lagrangian surfaces).

A more fundamental problem with this idea is the question of solid-body rotation. For a synchronous binary bigger than its second Lagrangian surface, the centrifugal potential dominates, and the surfaces of constant potential (normally the level surfaces) are unbound. Any star containing a double core must, therefore, be rotating *slower than synchronously* in its outer layers. I shall leave it to others to estimate the time the churning going on inside such an object would take to dissipate the angular momentum of the encased core. However, in this case, second-order effects of rotation cannot dominate the light or radial velocity of the star. Instead it must be subject to a nonradial pulsation driven by the encased binary. In this regard it is no different from other nonsynchronous binaries, such as the components of eccentric systems discussed by Wilson (1979).

Now, in the remote possibility that binaries with an encased core can actually exist long enough for us to observe them, just what might they look like? We can make some simplifying assumptions that let us calculate light and velocity curves for such an object. As a first stab, I have considered a system that is not rotating at all. This eliminates the centrifugal potential and makes the gravitational potential simply

$$\Omega = 1/r_1 + q/r_2, \quad (1)$$

where  $r_1$  and  $r_2$  are the distances of some point from the two components of the binary, the origin of coordinates is the binary's center of mass, and  $q = M_2/M_1$  is the mass ratio of the system. The surface corresponding to an equipotential of this equation is shaped roughly like a prolate spheroid for equal masses ( $q = 1.0$ ) and a large enough radius. For other mass ratios, the shape is more distorted and becomes rather biological at smaller radii. We can calculate the light and velocity curves for such objects if we make an assumption about how surface brightness changes over the star. I have made the simplifying assumption that the surface is always given by an equipotential, that this figure rotates with the binary system (i.e., has no phase lag), and that the surface brightness is determined by limb and gravity darkening as in a normal synchronously rotating binary. Actually, the surface brightness would be determined by the driven pulsation, so these assumptions are clearly bogus. However, they do give us a way to estimate the effect of having an encased binary as a star's core, at least to first order.

For these assumptions we may calculate the pulsational velocities, assumed radial, by comparing points equal distances apart in azimuth and dividing by the time it takes the binary to rotate that amount. Since in the calculations, we divide the star's



$g = 0.32$ . It should be possible to constrain the gravity darkening of convective stars much better in future with precise radial velocities for more eclipsing stars.

Tidal distortion should give a variable  $v \sin i$  for a binary component because the disc is actually broader at some phases than at others. We saw this effect in UU Cnc (Eaton et al. 1991). The new data for 5 Cet provide a further test, and I have applied it by looking at composite spectra of 5 Cet at the two conjunctions and at the two quadratures. First, blends of Fe lines near 6575 and 6594 Å, which can give a very good measure of  $v \sin i$  without numerically measuring and calibrating line widths (Eaton 1990), are somewhat sharper at the conjunctions than at the two quadratures. Second, widths of Gaussians fit to cross-correlation functions (of a list of strong solar absorption lines with the observed spectrum; Eaton & Williamson 2007) for these four composite spectra are about 5% broader for the quadratures than the conjunctions. See Table 2 for the details. We may simulate this effect by calculating synthetic spectra and measuring the variation of line width with phase, and I have done so for a rough model for the system. This model assumes the visible giant star is a contact component with  $x = 0.70$  and  $g = 0.32$ , and that  $K_1 = 23.9 \text{ km s}^{-1}$  and  $q = 0.8$ . I used a spectrum of  $\delta$  Eri (K0 IV) from the National Solar Observatory covering the wavelength range 6400–6480 Å to represent the cool component in this calculation, but the resolution is similar to that in the AST observations. The calculated line widths (col. [3] of Table 2) have roughly the same phase dependence as shown by the observations but are about 10% narrower. We can make them as large as the observed widths by reducing the mass ratio to about  $q = 0.7$ , thereby raising  $K_1 + K_2$ . In any case, a mass ratio greater than 1.0 (unseen star more massive than the contact component) gives profiles much narrower than observed.

#### 4. SUMMARY AND DISCUSSION

The existence of precise radial velocities for long-period binaries, made possible by a dedicated instrument on a robotic telescope, has made it possible to demonstrate the distortion of the radial-velocity curves of close binaries predicted by Sterne (1941). The distortions depend on a binary's inclination and mass ratio in ways that let us estimate these properties in favorable circumstances. Given the nature of this velocity-curve distortion, we must conclude that there is no credible evidence for orbital eccentricity in binaries with contact components. Furthermore, the distortion observed in three systems requires the stars to be gravity darkened, probably somewhat more than expected from Lucy's (1967) theory but much less than expected for stars with radiative envelopes.

In addition to the foregoing substantive results, I have also (1) detected the variation of  $v \sin i$  caused by tidal distortion in 5 Cet and used it to derive an improved mass ratio,  $M_{\text{unseen}}/M_{\text{gK}} \approx 0.70$ , (2) estimated somewhat improved geometrical properties for AX Mon, and (3) explored what properties a binary encased in a common envelope would have if we ever were to detect one.

##### 4.1. A Caveat about Limb Darkening

The greatest uncertainty in the results of this paper, as well in many others of the same genre, comes from the effect of limb darkening. This uncertainty would impact both papers analyzing light curves and those based on rotational line broadening or Doppler images. Such analyses usually parameterize the emergent intensity as a function of the angle between the surface normal and the line of sight, more specifically on  $\mu$ , its cosine.

The simplest formulation, a linear dependence on  $\mu$ , follows from the stratification of a simple hydrostatic atmosphere in radiative equilibrium. Other more complicated formulations are possible with increasingly less intuitive mapping into the atmospheric structure. Eventually, these effectively seem to become mere fitting schemes (e.g., Brown et al. 2001). Alternatively, one may apply a calculated intensity directly, perhaps by looking it up in a table of  $I_\lambda$  vs.  $\mu$ .

Theoretical model atmospheres predict levels of limb darkening that may be applied in analyses of binary stars and other problems. Such analyses usually assume limb-darkening coefficients from a few standard theoretical lists (e.g., Al-Naimy 1978; Claret & Gimenez 1990). Normally, such models assume plane-parallel stratification in hydrostatic equilibrium, basically the physics we apply to the Sun. A good example is the grid of atmospheres Kurucz calculated with his program ATLAS (e.g., Kurucz 1995). A recent alternative set of models is NextGen (Hauschildt et al. 1999a, 1999b). This latter grid ought to give a superior interpretation of our standard physical assumption; for example, by calculating line blanketing in the blue—UV through opacity sampling. Limb darkening from NextGen, especially from the spherical models for giants and supergiants (Claret & Hauschildt 2003), is quite different than from plane-parallel models. It tends to be much more pronounced, with the intensity dropping to zero well before the limb (see Orosz & Hauschildt 2000, their Fig. 4).

If real stars really have the limb darkening of these spherical NextGen models, there would be systematic errors in a wide range of observational results for giants and supergiants. For instance, it would make direct measurements of stellar diameters systematically small, leading to effective temperatures that are too high. This biases the Barnes-Evans relation (Barnes & Evans 1976), used extensively to derive angular diameters for cool stars and otherwise calibrate the flux-temperature relationship. This effect might be tested by analyzing the consistency of temperatures derived by fitting spectral energy distributions (e.g., Bertone et al. 2004) with those derived from measured bolometric fluxes and angular diameters. It would likewise bias rotational velocities from line broadening and a wide range of properties of binaries derived from such broadening such as mass ratios of noneclipsing systems, as discussed by Orosz & Hauschildt (2000) and Shahbaz (2003). It would also bias the results for *eclipsing* systems by making it harder to fit the eclipse shapes and ellipsoidal variation simultaneously. Furthermore, it might well bias the Doppler images of spotted stars, if the effects are as extreme as calculations of Claret & Hauschildt (2003, see their Fig. 2) imply. In our present analysis, the effect of the nontraditional limb darkening of NextGen models would likely be to increase the amplitude of the velocity variation somewhat, allowing for a lower gravity darkening. We can see this effect by making the linear limb-darkening coefficient  $x > 1.0$  and truncating the intensity at zero when it becomes negative near the limb. Such a calculation does increase the deviation from a sinusoidal velocity curve.

Now then, how much do we have to worry about the existing body of lore for stellar astronomy? Perhaps not as much as one would fear. There are few highly precise determinations of limb darkening for cool stars other than the Sun, and that is especially true for giants and supergiants. However, one reliable determination for a K giant (Fields et al. 2003) finds limb darkening much less extreme than predicted by NextGen models. Predictions of ATLAS were somewhat closer, but did not fit as well as one might hope. Fields et al. suspected unmodeled physical effects are to blame for this discrepancy, inasmuch as there are obvious effects in atmospheres beyond our current understanding. None of these models, for instance, incorporate the inhomogeneities

that we expect in all stellar atmospheres on the basis of solar granulation and Ayres's (e.g., 2002) work on CO. Furthermore, all these calculations necessarily use very simplistic models for turbulence, whatever that really is, and for convection. At this point, it seems the theoretical limb darkening of NextGen models is not tested well enough for us to worry seriously about its effect on binary-star analyses, but that we as a community should take it seriously enough to look for more tests, especially for the supergiants.

I would like to thank Peter Wood for stimulating me to think about this problem, for giving me a key reference to work on the long-period variations of AGB stars, and for clarifying some details of his work on rotating spheroids. This research used the SIMBAD database and was supported by NASA grants NCCW-0085 and NCC5-511 and by NSF grants HRD 9550561 and HRD 9706268.

*Facilities:* TSU:AST

#### REFERENCES

- Al-Naimy, H. M. 1978, *Ap&SS*, 53, 181  
 Ayres, T. R. 2002, *ApJ*, 575, 1104  
 Barnes, T. G., & Evans, D. S. 1976, *MNRAS*, 174, 489  
 Bertone, E., Buzzoni, A., Chávez, M., & Rodríguez-Merino, L. H. 2004, *AJ*, 128, 829  
 Brown, T. M., Charbonneau, D., Gilliland, R. L., Noyes, R. W., & Burrows, A. 2001, *ApJ*, 552, 699  
 Claret, A. 2000, *A&A*, 359, 289  
 ———. 2003, *A&A*, 406, 623  
 Claret, A., & Gimenez, A. 1990, *A&A*, 230, 412  
 Claret, A., & Hauschildt, P. H. 2003, *A&A*, 412, 241  
 Drekas, A., Kiss, L. L., Bedding, T. R., Kjeldsen, H., Lah, P., & Szabo, Gy. M. 2006, *ApJ*, 650, L55  
 Eaton, J. A. 1990, *Inf. Bull. Var. Stars*, 3460, 1  
 Eaton, J. A., & Barden, S. C. 1988, *Acta Astron.*, 38, 353  
 Eaton, J. A., Hall, D. S., & Honeycutt, R. K. 1991, *ApJ*, 376, 289  
 Eaton, J. A., Henry, G. W., Bell, C., & Okorogu, A. 1993, *AJ*, 106, 1181  
 Eaton, J. A., & Williamson, M. H. 2007, *PASP*, 119, 886  
 Elias, N. M., Wilson, R. E., Olson, E. C., Aufdenberg, J. P., Guinan, E. F., Guedel, M., van Hamme, W. V., Stevens, H. L. 1997, *ApJ*, 484, 394  
 Fields, D. L. et al. 2003, *ApJ*, 596, 1305  
 Hall, D. S. 1990, *AJ*, 100, 554  
 Hauschildt, P. H., Allard, F., & Baron, E. 1999a, *ApJ*, 512, 377  
 Hauschildt, P. H., Allard, F., Ferguson, J., Baron, E., & Alexander, D. R. 1999b, *ApJ*, 525, 871  
 Hinkle, K. H., Lebzelter, T., Joyce, R. R., & Feckel, F. C. 2002, *AJ*, 123, 1002  
 Kopal, Z. 1959, *Close Binary Systems* (London: Chapman & Hall)  
 Kurucz, R. L. 1995, *Kurucz CD-ROM 13, ATLAS9 Stellar Atmosphere Programs and 2 km/s Grid* (Cambridge: SAO)  
 Lucy, L. B. 1967, *Z. Astrophys.*, 65, 89  
 Lucy, L. B., & Sweeney, M. A. 1971, *AJ*, 76, 544  
 ———. 1973, *Observatory*, 93, 37  
 Miller, B., Budaj, J., Richards, M., Koubský, P., & Peters, G. J. 2007, *ApJ*, 656, 1075  
 Olivier, E. A., & Wood, P. R. 2003, *ApJ*, 584, 1035  
 Orosz, J. A., & Hauschildt, P. H. 2000, *A&A*, 364, 265  
 Russell, H. N. 1912, *ApJ*, 36, 54  
 Shahbaz, T. 2003, *MNRAS*, 339, 1031  
 Sterne, T. C. 1941, *Proc. Natl. Acad. Sci.*, 27, 168  
 Van Hamme, W., & Wilson, R. E. 1993, *MNRAS*, 262, 220  
 Wilson, R. E. 1979, *ApJ*, 234, 1054  
 Wilson, R. E., & Sofia, S. 1976, *ApJ*, 203, 182  
 Winn, J. N., et al. 2005, *ApJ*, 631, 1215  
 Wood, P. R., Olivier, E. A., & Kawaler, S. D. 2004, *ApJ*, 604, 800





