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V432 Per, a close binary star in poor thermal contact

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ABSTRACT

In a program instigated to understand close, possibly contact binary stars which appear to be in poor thermal contact, we have re-observed V432 Per both photometrically and spectroscopically. We conclude that the mass ratio is likely in the range $q = 0.30$–0.38, and the system clearly has a transit primary eclipse. The face of the less massive secondary towards the primary seems to be significantly hotter than expected. It was hoped that spectral line profiles would allow determination of this spot's radius ($r_{\rm spot}$), but the low mass of the secondary precluded this. With the existing observations, it is impossible to say whether this system is a true contact binary ($r_{\rm spot}$ 30°–90°), is semidetached ($r_{\rm spot}$ 90°–120°) or is marginally detached ($r_{\rm spot}$ 120°–150°). However, while it may be marginally in physical contact, it is clearly in poor thermal contact. With its short period and marginal contact, this system merits further attention as a potential transitional species in binary-star evolution.

Key words: binaries: close – stars: binaries: eclipsing – stars: individual: V432 Per.

1 INTRODUCTION

Binary stars evolving through mass exchange are expected to undergo temporary contact configurations or to evolve into stable contact binaries, depending upon their initial separation. Details of just how such a system evolves and what the outcome of the mass transfer would be depend on yet murky theories of luminosity transfer in a common envelope. Two such theories, which exist mostly as general frameworks, somewhat like Darwin's famous theory, are a Thermal Relaxation Oscillation (TRO) model (Lucy & Wilson 1979), an extension of Lucy's (1976) common convective envelopes, and a contact-discontinuity (DSC) model (Shu, Lubow & Anderson 1976).

There is a poorly understood group of binary stars either in contact or very close to contact but which have different eclipse depths, indicating that the two stars have substantially different temperatures. These EB-type light curves make the stars candidates for the broken-contact phase predicted by the TRO model. Relating these systems to the classical, presumably stable W UMa-type contact binaries is difficult because their fundamental physical properties are problematic – difficult to determine and thereby poorly known. Light curves alone leave ambiguities in the properties of such systems and their component stars, particularly in the mass ratio q and in the distribution of temperature (hence surface brightness) on the secondary star.

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These two ambiguities can be removed with spectra, which yield radial velocity curves and line profiles for the system. For this reason, we have undertaken a survey to determine unambiguously all the properties of several of these systems, and how they are related to the traditional contact binaries. The first of these is V432 Per. Maceroni & van't Veer (1993) first pointed out that the secondary component of such a system is well seen only at primary eclipse, and that all we really know about it (from the light curve alone) is that the anti-neck (180° from the neck) is substantially cooler and darker than the primary star.

V432 Per (GSC 02856−01647) was reported to be variable by Hoffmeister (1968) and subsequently observed photoelectrically by Agerer (1992), who corrected the orbital period to $0\overset{\rm d}{.}383\,312\,34$ and showed that the eclipses are total. Yang & Liu (2002; hereafter YL) obtained new CCD photometry but their light curves did not show the total secondary eclipse so obvious in Agerer's data either the system had changed or the photometry of YL was not of adequate quality. More recent CCD photometry by Lee et al. (2008; hereafter LEE) confirms the total eclipses, as does our own (see Section 4.1). YL solved their light curves, with their apparently partial eclipses, using the Wilson–Devinney code (Wilson 1994) and concluded that the primary eclipse is an occultation. LEE likewise fitted their light curves with the Wilson–Devinney code, proffering separate solutions for which they assumed primary eclipse is either an occultation or a transit. We can test these results by combining spectra (to define the line profiles and component velocities) with our new more precise photometry. Our photometry agrees with Agerer's in shape, and our Hα line profiles prove that primary minimum must be a transit, not an occultation.

Table 1. Times of Spectra for V432 Per.

HJD (mid) 240 0000+	Phase	Vr (prim) (km s^{-1})	HJD (mid) 240 0000+	Phase	Vr (km s^{-1})
53013.5813	0.760	132.4	53013.7657	0.270	5.3
53013.5923	0.789	122.2	53013.7767	0.305	7.1
53013.6042	0.820	120.5	53013.7901	0.334	23.8
			53013.8011	0.241	3.5

Table 2. Photometry Filter Characteristics.

Filter	*b*	*R*
Wavelength (Å)	4820	6000
FWHM (Å)	200	1500
Integration (s)	20	5, 8
Measurements	256	273

2 OBSERVATIONS

We observed V432 Per spectroscopically with the 2.12-m telescope at The Observatorio Astrofísico Guillermo Haro (OAGH) in Cananea, Sonora, Mexico on UT 2004 January 9 (JD 245 3013). The Boller and Chivens Spectrograph was used with the 830-line grating in second order, giving a dispersion of 0.31 Å pixel^{-1} on the Tektronix TK1024 CCD camera (24-m pixels). We made seven 15-min exposures (0.03 phase), which had a signal to noise of about 80. From line widths in the comparison emission spectra, the resolution was about 1.4 Å full width at half-maximum (FWHM). We also took a spectrum of HD 23167, a faint G star, which has Fe I lines comparable to those in the Sun. It would have to have instrumental broadening near FWHM = 2.3 Å if its intrinsic Hα line is comparable to the Sun's. However, some of this broadening is likely caused by partially resolved telluric lines. A spectrum for HD 3765, a single K2 V star with high proper motion, has roughly the right equivalent width of Hα (0.98 Å) for a K star but wings much too weak for V432 Per. The central depth of Hα in this star corresponds to an instrumental broadening near FWHM = 2 Å which was used in simulating the profiles. The final spectral resolution is about $\lambda/\Delta\lambda \approx 3500$.

Table 1 gives the phase coverage of our seven spectra. We attempted to derive radial velocities for the primary binary component from these spectra, using a He–Ne source for their wavelength calibration and measuring velocity shifts from cross correlation with a spectrum of HD 23167. The resulting velocities could not represent the orbital velocity of the primary star. They yield a velocity amplitude of only $K_1 = 62$ km s^{-1}, versus the 93 km s^{-1} implied by the masses that LEE assumed, which yields the implausibly low mass $M_1 = 0.59$ $M_{\odot}$. However, since we have the actual line profiles, we assumed various K_1's (i.e. masses) and a light-curve solution and found masses that fit the profiles (see Section 4.2).

Our spectra do not contain enough lines to derive a spectral type, but based on the strength of the Hα line, along with the $(B - V)$ colour of 0.62 given by YL, we judge the primary star to be roughly solar or slightly later. This allows us to estimate the temperature of the primary star for the modelling, which is in any case not very sensitive to the chosen value.

We also observed V432 Per photometrically with the Kuiper 1.55-m telescope of the Steward Observatory on Mt Bigelow AZ on UT 2006 December 25, 26 and 27 as well as 2007 January 8 (JD 2,454,094, 095, and 096, and 108). The Mont4K CCD[1] was used in direct mode with a Stromgren *b* and a broadband Harris *R* filter to determine the light curve. On-chip binning of 3 × 3 was used, which yields 0.45 arcsec pixel^{-1}, and a field of 10.2 arcmin. Table 2 summarizes the properties of these observations.

[1] The 'Montreal 4K' CCD Imager was made possible through the support of the University of Montreal and the Canadian Research Council.

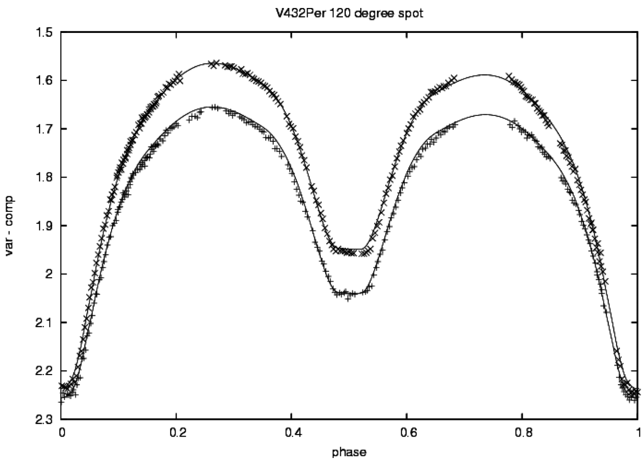

Figure 1. Fit of the model to the two light curves, in *b* (upper) and *R*. Observed magnitudes are the symbols; the continuous curves represent the solution given in Table 4.

Our comparison and check stars were the ones used by YL, viz. SAO 38621, which is only 0.08 mag bluer than V432 Per in $(B - V)$, and GSC 02856−01231. Both of these were in the same field as V432 Per, and, because of the similar colour of the comparison star, no colour terms were used in the reduction.

We reduced the images with IRAF[2] (Tody 1986): first using CCDPROC to apply the overscan, trim, zero-frame, and flat-field corrections, then APPHOT to derive magnitudes for the variable, comparison, and check stars. Next, the quantities (var-comp) and (comp-check) were formed, and the Heliocentric Julian Date was computed from UT in the header. The Ephemeris below was used to compute the phase. These data will be made available in the Journal of Astronomical Data.

Our new light curves (Fig. 1) show the same general shape as Agerer's, namely a pronounced phase of complete eclipse (occultation) at secondary minimum. There was also an asymmetry between the two elongations at this epoch.

3 EPHEMERIS

The literature contains a plethora of times of minimum for this star, and YL give those available to them in their table 2 as well as show an $(O - C)$ diagram in their fig. 1. YL analyse the timings in three ranges of date, for which they could define two periods: (1) 1958–1973, P1 based on photographic and visual photometry,

[2] IRAF is written and supported by the IRAF programming group at the National Optical Astronomy Observatories (NOAO) in Tucson, Arizona. NOAO is operated by the Association of Universities for Research in Astronomy (AURA), Inc. under cooperative agreement with the National Science Foundation.

(2) 1973–1990, no timings, and (3) 1991–2001, P2 (>P1 by 0.7 s), based on photoelectric (PE) and CCD photometry–13 timings in six observing seasons (91–92, 92–93, 94, 97, 98 and 2000–2001; YL's timings were in 1998).

We captured two minima in our photometry and have determined their times by reflecting the ascending branch on to the descending branch while adjusting the HJD of the fold to get the best match. We then averaged the times for the two filters to get primary minimum at HJD 245 4108.796 85 and secondary minimum at HJD 245 4094.806 75. These new times agree well with YL's ephemeris. None the less we have combined the existing PE and CCD times since 1990 with ours to obtain a new ephemeris used to phase our photometry and spectra:

$$\text{HJD Min I} = 245\,2897.5295(3) + 0.383\,3126(2)E, \qquad (1)$$

for which uncertainty in final digit is given in parenthesis. YL suggest that the period of V432 Per changed about 1973, and we concur that the period lengthened by about 0.7 s around that date. This new, longer period has remained constant since the analysis of YL.

LEE give a potentially more sophisticated analysis of the $(O - C)$ diagram, interpreting it as a combination of a continuous period increase, to give the overall parabolic shape, and a light-time effect in an eccentric 35-yr orbit, to explain the shorter term variations. This implies a more or less continuous transfer of mass between the components and agrees with the currently fashionable idea that having a third component in a wide orbit is a sink of angular momentum *required* to have a close binary in the first place (e.g. Rucinski, Pribulla & van Kerkwijk 2007).

These two approaches constitute fundamentally different ways of thinking about period changes in close binaries. YL's representation of them as essentially a number of discrete events assumes the notion developed by Hall (1990) in his study of period changes in close binaries that they are preponderantly episodic, possibly caused by some unknown phenomenon of magnetic cycles. On the other hand, LEE use entirely continuous processes that can be modelled mathematically. The stochastic approach, whether it supposes fluctuations in stellar structure from magnetic cycles or from episodic mass transfer, is theory in outline only without any truly hard predictions.

Which, if either, of these two approaches is correct? It would be dishonest to dismiss the episodic theories simply because we astronomers are too stupid to make predictions for them. Eventually, this star may give an answer, if effects of the 35-yr orbit repeat, but that will be after we have all passed from the scene. In the meantime, we must also keep in mind that the seemingly smooth period increase, supposedly caused by unidirectional mass transfer, may simply be the result of episodic transfer we cannot resolve, as per Hall (1990). We are thus left in a situation much like that of dealing with global warming in which we would like to isolate the effect of increasing CO_2 concentration but cannot unequivocally disentangle it from natural fluctuations in Earth's atmosphere, unmodelable but known to exist.

4 PHOTOMETRIC AND SPECTROSCOPIC SOLUTION

4.1 The light-curve solution

We have solved the light and velocity curves with the Wilson–Devinney Code (2003 version; see Wilson & Devinney 1971; Wilson 1979; and Wilson 1990), which allows spots at arbitrary positions on the components of a contact binary. This is important because the light curves of V432 Per and similar stars tend to be asymmetric as though parts of the surface are hotter or cooler than expected in the standard picture of a binary system. In particular, phases either side of 0.5 are brighter, when the side of the smaller secondary star faces the larger primary star are most exposed. This implies the secondary has either a bright spot on its neck facing the primary, or a dark (cool) spot on its rump facing away. The star also shows that the O'Connell Effect (O'Connell 1951; Milone 1968) is slightly fainter (by about 0.02 mag at this epoch) at phase 0.75 than at phase 0.25. We modelled this effect with a dark spot on the trailing side equator of the primary with a temperature about 1 per cent cooler than the rest of the star in all cases.

Among the models we have had to consider are contact models in which the binary is enclosed in a common envelope between the inner contact surface (Roche lobes) and the second contact surface. The degree of contact is usually specified by a filling factor, f, defined in Wilson's terminology as the fraction of the difference in gravitational potential between the inner and outer contact surfaces that the star's surface lies above the inner contact surface. A small f implies the star is just overfilling its Roche lobe, and this is the usual case for contact binaries. If f becomes negative in a solution, the 'contact' component has shrunk beneath its Roche lobe. Other properties defining the model are (1) the mass ratio, q, which determines the eclipse depths in a totally eclipsing system, (2) the orbital inclination, i, (3) the assumed, fixed temperature of the primary component, T_1, (4) the adjusted temperature of the secondary component, T_2, (5) the various properties of spots, namely their locations, sizes and temperatures, (6) a multiplier to fit the overall level of the intensities and (7) coefficients for limb darkening, gravity darkening and the reflection effect, all of which are usually assumed to have some theoretical values.

Although we investigated dark spots on the backside of the secondary, the better solutions all involved putting a bright spot on the neck of the secondary facing the primary. The data give a great deal of leeway in choosing the size and temperature of this spot, so we have explored the eight families of solutions listed in Table 3 in which we fixed the spot size and solved for the other parameters. For a fixed spot size and location, we vary q, f, i, T_2 and $T_{\rm spot}$. The relative quality of the fits can be judged from the rms residuals in Column (4) of Table 3. We have chosen the model with $r_{\rm spot} = 120^\circ$ as providing the best fit; Table 4 gives detailed properties for it. The solid line plotted in Fig. 1 is this solution.

The geometrical elements seem to be very well determined as judged by the quoted formal errors, but these must be taken with a grain of salt (Popper 1984). To get a better idea of the true errors in this sort of solution, we tried using a different program (Eaton et al. 1993) than the Wilson-Devinney code and fitted the extra brightness near phase 0.5 with a highly elevated albedo for the secondary, essentially the same as using a bright spot on the neck. In this case, the solution tended towards marginal contact ($f = 0.005$) with a mass ratio $q = 0.356 \pm 0.002$ and $i = 84\overset{\circ}{.}77$.

4.2 A spectroscopic test

Line profiles can provide an excellent way of testing and constraining the light-curve solution. Given a light-curve solution it is easy enough to calculate a line profile (e.g. Eaton, Hall & Honeycutt 1991; Eaton et al. 1993; Eaton & Henry 2007). The scale of the profile depends on the total velocity amplitude of the orbital motion, $K_1 + K_2 = K_1(1 + 1/q)$, and the calculated profile gives a good test of the mass ratio even if one cannot measure velocities for the

Table 3. Summary of models with different size spots on the secondary.

No.	Spot Radius (°)	*Fill factor*	$\mid \ell_{obs} - \ell_{cal} \mid_{ave}$ (max light)	T_2 (K)	T_{spot} (K)	$L_2/(L_1 + L_2)$
1	30	0.1037 (c)	0.0370	5025	6533	0.171
2	60	0.0835 (c)	0.0170	5058	5645	0.171
3	90	0.0474 (c)	0.0150	5047	5511	0.171
4	110	−0.0396 (s-d)	0.0141	4989	5493	0.156
5	120	−0.0673 (d)	0.0141	4903	5486	0.156
6	130	−0.0808 (d)	0.0144	4748	5474	0.156
7	140	−0.1218 (s-d)	0.0163	4525	5435	0.105
8	150	−0.1297 (s-d)	0.0170	4155	5343	0.085

Note. Column 3, 'c' stands for contact, 'd' for detached, and 's-d' for semidetached. In the case of s-d, the fill factor is for the primary.

Table 4. Solution for $r_{spot} = 120°$.

Parameter	Value
	Derived
q	0.303 ± 0.001
f_1	-0.0673 ± 0.01
f_2	-0.0230 ± 0.01
i	$81.°64 \pm 0.23$
Ω_1	2.4860 ± 0.003
Ω_2	2.4776 ± 0.004
T_2 (K)	4903 ± 10
T_{spot} (K)	5486 ± 7
r_{spot}	120°
K_1 (km s^{-1})	74–87
	Assumed (Theoret)
T_1 (K)	5800
x_R	0.56
x_b	0.72
g	0.32
A_{Bol}	0.50
	Inferred
$r_{side,1}$ (pri)	0.488 (1.25 $R_\odot$)
$r_{pole,1}$ (pri)	0.455
$r_{pole,2}$ (sec)	0.276 (0.71 $R_\odot$)
$r_{pole,2}$ (sec)	0.265
a ($R_\odot$)	2.59 (K1 = 79 q = 0.303)
M_1 ($M_\odot$)	1.22
M_2 ($M_\odot$)	0.37

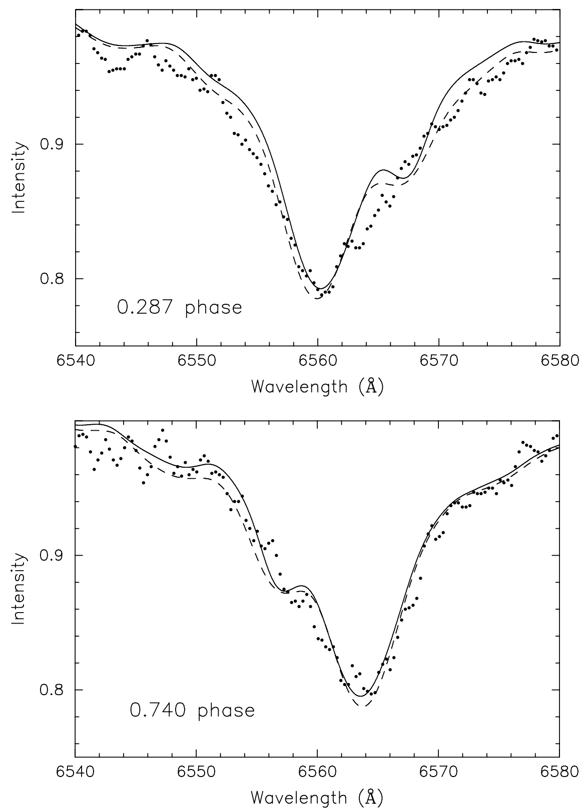

Figure 2. Fit of the model to an Hα spectrum at two representative phases. Observed intensities are the dots. The curves show profiles calculated for two solutions, solid, our solution of Table 4; dashed, LEE transit. Our solution for marginal contact with elevated reflection effect gives similar results.

secondary directly. One caveat, however – the profile we are fitting is for Hα, which has strong damping wings in the typical F or G dwarf and which may be 'filled in' by emission in chromospherically active stars and some contact binary components (Barden 1984a,b). Hα, however, is the only line available strong enough to show a well-defined profile in cool contact binaries. There is actually some evidence that the core of Hα in V432 Per may be partially filled in. Equivalent widths of Hα in the profiles shown in Fig. 2 are about 1.88 versus 2.43 Å in HD 23167 and the Sun. We have therefore used a solar spectrum as a template with the core of Hα reduced by 0.55 Å in EW in our simulations of the spectrum.

To apply the Hα profile as a test of models, we have formed composite spectra for two representative phases, $\phi = 0.287$, consisting of four spectra for the phase range 0.228–0.347, and $\phi = 0.790$, consisting of three spectra for the range 0.747–0.833. This combination reduces somewhat the noise inevitable in observations of such a faint star, but at the expense of phase resolution. We have reduced it further by performing a running three-point average of the fluxes, but at the expense of spectral resolution. The profiles at these two phases show in their opposite asymmetries the effect of the blending of the lines of the two binary components. They also show in their overall velocity shifts that the more massive star eclipsed at primary minimum is the brighter, hence larger component. Primary minimum is a transit. We estimate from simulations with different velocity amplitudes that the total orbital velocity is in the range $K_1 + K_2 = 320$–375 km s^{-1}.

Our calculated profiles fit the observed ones reasonably well, as we can see in Fig. 2. There are obvious ways in which they do not fit, however. The profile for the primary is too narrow and would be much more so if we had included the whole core in our template. The fit would be much better with instrumental broadening of FWHM = 3–4 Å, but that is ruled out by observations of single stars. That the line of the secondary is so weak favours the small mass ratio we have found.

5 DISCUSSION

We now have four detailed analyses of the light variation of V432 Per from which a consensus model emerges. The system seems to be in marginal contact with its Roche lobe, probably very slightly semidetached or detached. The details here depend how one resolves departures of the light curve from the standard Roche model. YL used dark spots on one or other component, finding the larger star to be the cooler component (occultation at primary minimum). LEE used a dark spot on the primary component, whichever one it was, preferring a transit eclipse at primary minimum. We have used a bright spot on the secondary component, which is the cooler less massive one in all our solutions; and in a separate solution, we simulated this effect by using an elevated albedo for the secondary.

Both the shapes of the light curves and the Hα line profiles prove that primary eclipse is a transit, the more massive, hotter star being eclipsed by its less massive cooler companion. The occultation solutions of YL and LEE are thus irrelevant. For the transit solutions, with their variety of assumptions about the cause of the elevated sinθ wave in the light curve, the mass ratio falls in the range $q = 0.30$–0.38. Some of this range comes from LEE's allowing third light, which would require a larger value of q.

The system has the basic properties of Binnendijk's (1970) A-subclass of contact binaries in terms of its period and a likely late-F spectral type. However it does not have the rather deep common envelope and minimal difference in temperature between the components of the typical A-type system. In fact, the best solutions seem to be very slightly detached, perhaps semidetached, although it is not clear that the Roche model can restrict the geometry of a system, this complicated well enough to discriminate among these possibilities. There may be some sort of mass exchange between the components as shown in the period increase. However, assuming this system is an example of Case-A evolution (Paczynski 1971) and that one star has reached its Roche lobe, we cannot even tell which one it is. There are at least three intriguing possibilities for the evolutionary state of this star. (1) It may be the closest Algol binary yet known, with the cool secondary formerly the more massive primary and the present primary now rotating more rapidly than synchronously (cf. Van Hamme & Wilson 1993). This might explain the broad Hα line profile and the period increase via mass exchange. (2) It could represent an erratic cyclical stage of mass transfer in the formation of a stable contact binary with the primary having reached its Roche lobe. In that case, we can speculate that rapid swelling of the secondary would choke off the mass transfer and might lead to episodic transfer as the two stars groped their way towards thermal equilibrium. (3) Then it might be a case of the long-sought thermal relaxation oscillations of Lucy & Wilson.

We hate to fall back on the old cliche of needing more and better observations, but this is truly a system that could benefit from higher resolution and signal-to-noise spectra to define its line profiles. Such profiles, if extended to an ensemble of lines besides Hα, would detect the secondary component unequivocally and let the profiles of both stars determine whether rotation is synchronous and where the spots are located. Spectra with signal to noise greater than 100 and resolution of 0.5 Å would resolve the final ambiguity of this star, that is the size of the hot region on the secondary and therefore whether the system is in physical contact or not.

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