Multiperiodicity in Five Small-Amplitude Pulsating Red Giants

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ABSTRACT. We report multiperiodicity in five small-amplitude pulsating red giants: RZ Ari, V523 Mon, BC CMi, UX Lyn, and FS Com. For each of these stars, two or three periods recur in each season of our 5000 day database of *V* observations. The periods and their ratios are consistent with low-order radial pulsation modes. The amplitudes of the modes change significantly on timescales of 1–5 yr; specifically, the amplitudes of the dominant periods vary on timescales of 2000–3500 days. Most often, the amplitudes of the modes in a given star rise and fall in unison.

1. INTRODUCTION

Giants cooler than K5 III are almost all variable in brightness (Eyer & Grenon 1997; Grenon 1993; Henry et al. 2000; Jorissen et al. 1997). K5–M9 giants make up about 10% of the stars in the Bright Star Catalogue. Generally, the cooler the star, the larger the *V* amplitude and the longer the period. The coolest stars are the relatively rare large-amplitude, long-period Mira stars. The present paper deals with the much more numerous small-amplitude pulsating red giants (SAPRGs). These stars have *V* amplitudes of 0.05–1 mag or more. The dominant periods of the SAPRGs are consistent with low-order radial pulsation modes, according to the small sample studied by Percy & Parkes (1998), who compared observed *Q*-values with theoretical ones. The *Q*-value, or pulsation constant, is the period in days times the root of the mean density in solar units. Percy & Bakos (2003) recently analyzed a sample of 76 SAPRGs using the same method. They found that about 30% of the stars pulsate in the fundamental (F) mode, about 50% in the first overtone (1H), about 10% in the second overtone (2H), and about 5% in the third overtone (3H). The very high overtone periods in a small number of SAPRGs in the *Hipparcos* Catalogue (Koen & Laney 2000) are almost certainly spurious (Percy & Hosick 2002).

Many SAPRGs do, however, have long secondary periods. About a third of the stars in the Percy & Bakos (2003) sample have such periods; the median ratio of the long secondary period to the dominant radial period is about 10. Larger amplitude semiregular variables also display long secondary periods (Houk 1963; Kiss et al. 1999), also about 10 times the dominant radial pulsation period.

Stars that are pulsating in two or more modes—multiperiodic

pulsators—provide information about the modes and also about the physical properties of the stars (Jorgensen & Petersen 1967). Multiperiodicity is observed in a wide variety of pulsating stars, including Cepheids and RR Lyrae stars. Kiss et al. (1999), using visual data, have reported multiperiodicity in several dozen larger amplitude semiregular variables. Wood (2000) has studied pulsating red giants in the LMC and has identified several sequences in the *K* magnitude–log (period) diagram, and these can be interpreted as stars pulsating in different radial modes.

Percy, Wilson, & Henry (2001, hereafter PWH) have recently published 5000 day light curves and periods of about three dozen SAPRGs. The 200 day light curves of some of these SAPRGs show evidence of possible multiperiodicity. We analyzed five of these stars—RZ Ari, V523 Mon, BC CMi, UX Lyn, and FS Com—that appeared to have periods of 50 days or less. We chose shorter period variables so we could determine reliable periods from individual seasons of data.

2. METHOD

We began by computing power spectra of the first and second halves of the 5000 day *V* data sets of PWH, using the Fourier/ CLEAN program TS (Foster 1995), which is available on the Web site of the American Association of Variable Star Observers (AAVSO), to see whether there were periods that occurred in both halves of the data set. Indeed, there were (§ 3). We noted the presence (as expected) of alias peaks, due to the seasonal gaps in the data.

We then analyzed individual seasons of data on each star, noting the two or three most prominent peaks. These power spectra would not be affected by aliasing. The first third of the data is sparser than the last two-thirds, and there was one interval in which the telescope was closed for repairs for about 250 days. We checked, using simulations, to make sure that using 200 day seasonal data sets would not have a systematic effect on our period determinations.

For each star it was observed, using power-period plots (Figs. 1–5) and stem-and-leaf plot analysis (Tukey 1972), that two

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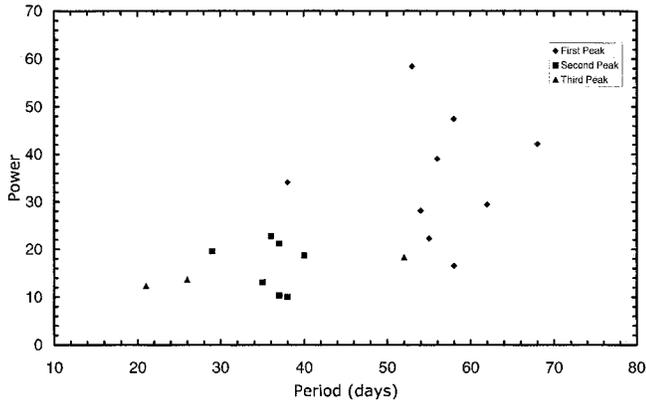
FIG. 1.—Power-period plot for the dominant periods in the power spectra of each season of data for RZ Ari.

FIG. 3.—Same as Fig. 1, but for BC CMi.

or three periods recurred each season. (The stem-and-leaf plot provides slightly more information than a histogram. It was invented by John Tukey; in it, microdata replace the information-empty bars of a traditional histogram. Tukey [quoted in Tufte 1990] states, "If we are going to make a mark, it may as well be a meaningful one. The simplest—and most useful—meaningful mark is a digit.") The average values of these periods were determined for each star. These periods, and their ratios, were compared with theoretical values (Fox & Wood 1982; Ostlie & Cox 1986; Xiong, Deng, & Cheng 1998), and they are consistent with low-order radial pulsation modes; these periods could be used to determine Q-values, and hence pulsation modes, as described in the previous section.

Having discovered evidence for multiperiodicity in these stars, we then asked the question: How do the amplitudes of the modes vary with time? Are the amplitudes constant? Do they rise and fall in unison? Do they exchange energy such
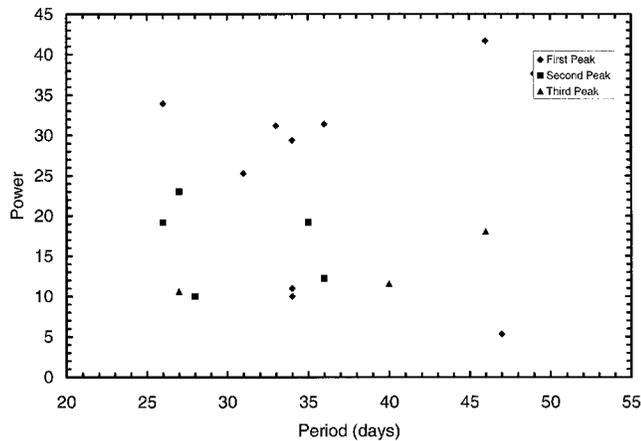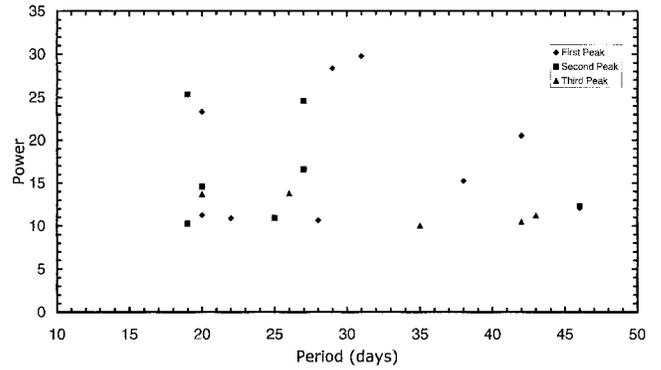
that the total pulsation energy is constant with time? Or is the situation more complex?

For each of the five stars, the seasonal light curves were fitted using the two or three periods determined above, using the Period98 program (Sperl 1998) available on the Web site of the University of Vienna. The amplitudes of each period for each season were determined and plotted against time (Figs. 6–10). Recall that the data for the first two or three seasons are slightly sparser than the rest.

The fits of these periods to the data were not perfect; there were residuals of up to a few hundredths of a magnitude. This is not unexpected. Our results showed that the amplitudes varied significantly from year to year—occasionally from their minimum to their maximum value—so it is to be expected that they will vary during each season. The fits assume constant amplitudes. As noted in § 3, there are also other processes that could contribute to the variability.
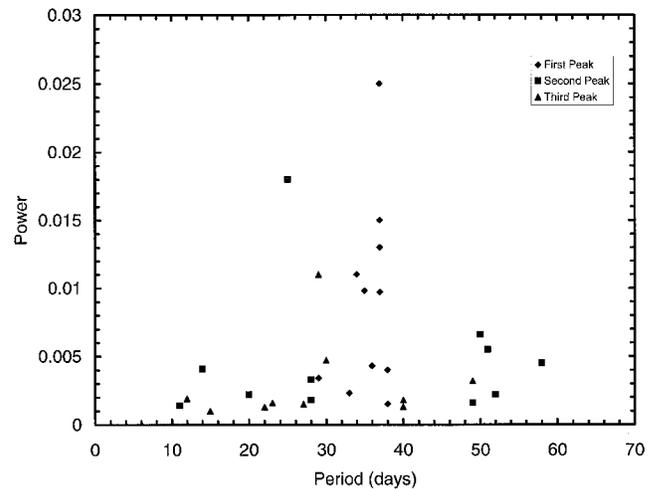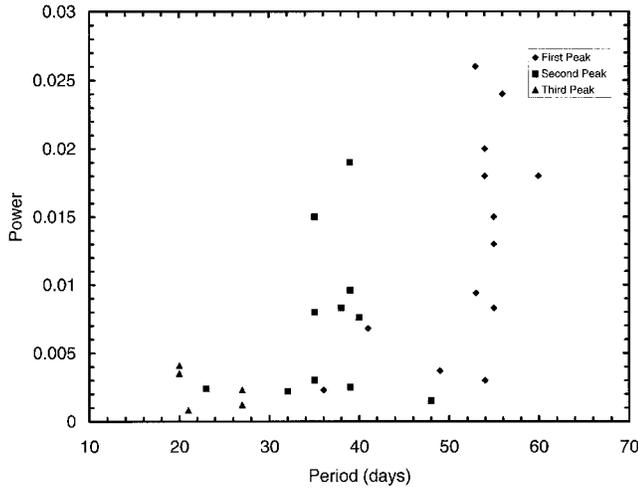

FIG. 2.—Same as Fig. 1, but for V523 Mon.

FIG. 4.—Same as Fig. 1, but for UX Lyn.

FIG. 5.—Same as Fig. 1, but for FS Com.

FIG. 7.—Same as Fig. 6, but for V523 Mon.

3. RESULTS

Figures 1–5 show the power and periods of the principal modes, as seen in the different seasons. The HWHM of the peaks was typically 3%–5%. Clusters of recurring periods can be seen. In order of decreasing amplitude, they are as follows: for RZ Ari, 56 and 37 days; for V523 Mon, 34, 47, and 27 days (the latter two approximately equal in average amplitude); for BC CMi, 27, 20, and 42 days; for UX Lyn, 36 and 51 days, with a possible shorter period around 27 days; for FS Com, 54 and 37 days, with a possible shorter period around 25 days. The longest period is not always the dominant one.

The periods of the dominant modes were also determined independently using the least-squares program Period98, and similar periods and clustering were found. In one or two cases,
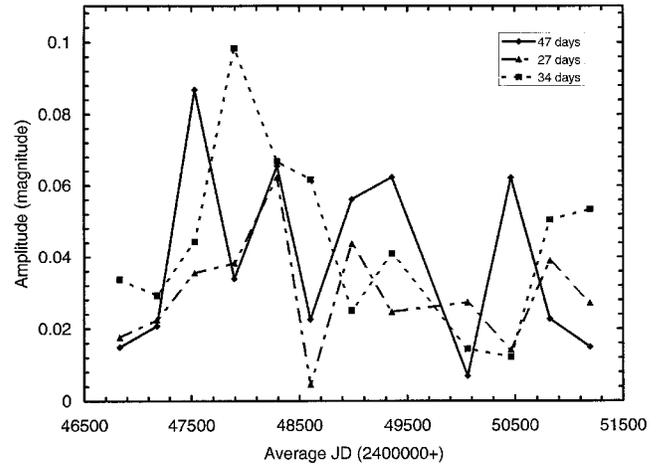
the least-squares program, when applied to a season of data, would choose a period of 150 days or longer to represent the longer term trends in the data. However, since the seasonal data sets were only 150–200 days long, these longer timescales are not "periods."

The results are summarized below and in Table 1. The temperatures, radii, Q-values, and mode identifications are taken from Percy & Bakos (2003). In Table 1, P_s is the mean seasonal period, P_h is the mean period from the two halves of the data, the rank is on the basis of the mean amplitude of the modes, the mode is determined from the Q-value and the period ratio, and R_{obs} and R_{model} are the observed period ratios and the range of possible ratios from different theoretical models, respectively.

RZ Ari.—Q-values 0.028 (2H) and 0.042 (1H); the period ratio favors F and 1H, but the disagreement can be avoided by adjusting the mass.
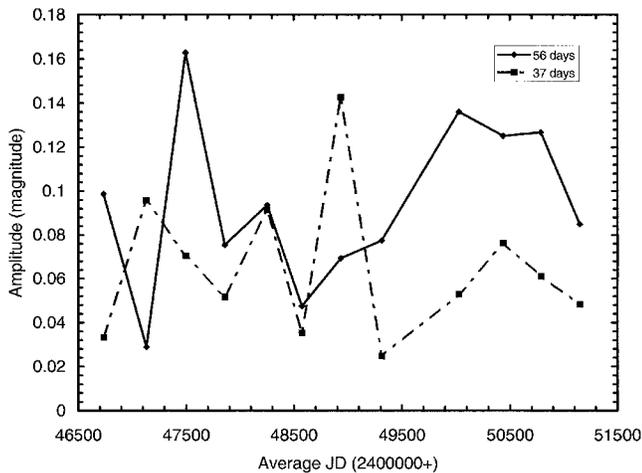

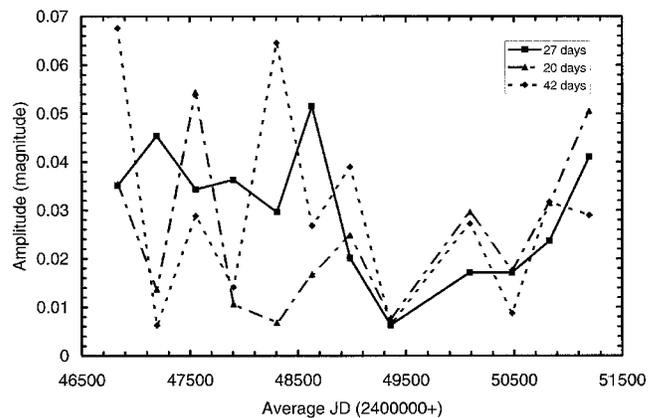FIG. 6.—Time variation of the seasonal amplitude of each of the dominant periods for RZ Ari.

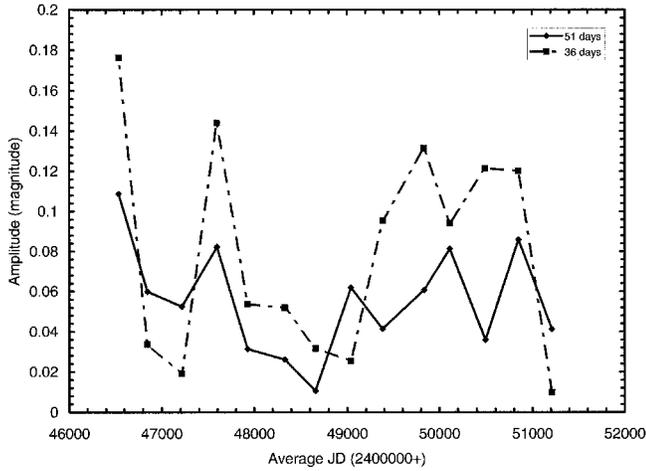FIG. 8.—Same as Fig. 6, but for BC CMi.
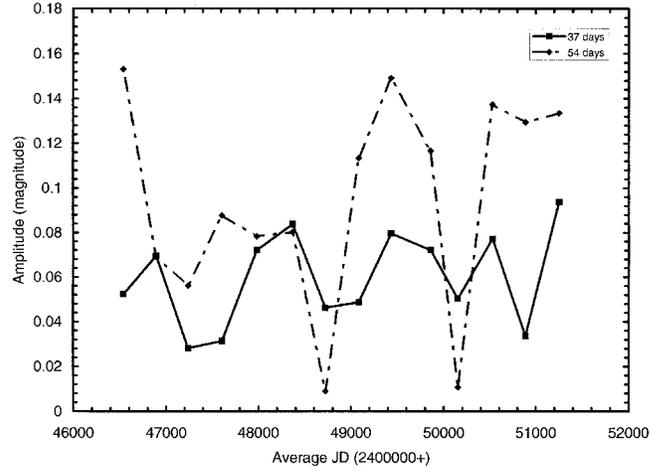
FIG. 9.—Same as Fig. 6, but for UX Lyn.

FIG. 10.—Same as Fig. 6, but for FS Com.

V523 Mon.—Q-values 0.029 (3H), 0.037 (2H), and 0.051 (1H); the period ratios are consistent with these modes.

BC CMi.—Q-values 0.033 (2H), 0.045 (1H), and 0.069 (F); the period ratios are consistent with these modes. The power spectra of the first and second halves of the data set suffer from severe aliasing, but there are groups of peaks around 19–21, 26–30, and 40–50 days.

UX Lyn.—Q-values 0.029 (2H) and 0.041 (1H); the period ratio is consistent with these modes. The longer period is weak in the first half of the data set. There may be a weak period of about 29 days in both halves.

FS Com.—Q-values 0.031 (2H) and 0.045 (1H); the period ratio is consistent with these modes. The 37–38 day period is strongly aliased in the power spectra of the two halves of the data set.

The amplitudes definitely change significantly from season to season. Sometimes, they change from a high to a low value in consecutive seasons; see the first three seasons of RZ Ari, for instance. At other times, it appears that they rise or fall consistently over 3–5 yr. The timescales associated with the rise and fall of the amplitude of the *dominant* periods (given in parentheses) are for RZ Ari (56 days), 3500 days; for V523 Mon (34 days), 3500 days; for BC CMi (27 days), 3000 days; for UX Lyn (36 days), 2500 days; for FS Com (54 days), 3500 days. The limited length of our data set (about 5000 days) prevents us from knowing whether these timescales persist with time. In four of the five stars, there are one or two epochs when all of the modes have lower amplitude: BC CMi, JD 2,449,400; V523 Mon, JD 2,447,000 and 2,450,000; UX Lyn, JD 2,448,600; FS Com, JD 2,447,200. In these stars, the amplitudes of the different modes fell in unison before these dates and rose in unison after. From a purely statistical point of view, however, there were almost as many instances of the amplitudes of the different modes changing in the opposite sense from one season to the next as there were changing in the same sense. Many of these instances, however, were during the first seasons when the data were sparse.

4. DISCUSSION

We have detected two or three recurring periods in each of the five SAPRGs. The reality of these periods is demonstrated by (1) their recurrence in almost every season and (2) their presence in the power spectra of both the first and second halves of the data set. We interpret these periods as low-order radial pulsation modes, on the basis of both their Q-values and their period ratios.

For four of the five stars, the period ratios were consistent with mode identifications determined from Q-values, assuming masses of 1.4 times solar. The Q-value, however, is sensitive to the mass. The fact that the mode identifications do not agree for one star, RZ Ari, could be explained by the choice of assumed mass. Dumm & Schild (1998) have presented a mass-radius relationship for these stars, although there is no a priori reason why every star should conform to this relation.

We must keep in mind that the two-period and three-period fits still left significant residuals. As mentioned above, this must certainly be partly due to changes in the amplitudes during

TABLE 1
PROPERTIES AND PERIODS OF PROGRAM STARS

Star	T_e	$R/R_\odot$	P_s	P_h	Rank	Mode	R_{obs}	R_{model}
RZ Ari	3305	145	37	37.7	2	2H	...	...
			56	56.5	1	1H	1.50–51	1.42–51
V523 Mon	3439	110	27	26.0	2	3H	...	...
			34	34.1	1	2H	1.26–30	1.30:
			47	45.6	2	1H	1.35–38	1.42–50
BC CMi	3495	71	20	20:	2	2H	...	...
			27	28:	1	1H	1.35–38	1.38–49
			42	45:	3	F	1.56–60	1.58–67
UX Lyn	3328	133	36	37.2	1	2H	...	...
			51	51.3	2	1H	1.38–42	1.42–51
FS Com	3418	136	37	38.2	2	2H	...	...
			54	55.4	1	1H	1.45–46	1.40–51

individual seasons. One possible approach would be to use a period-determination method that could cope with such changes; we have made preliminary investigations of the use of wavelet analysis for these stars (Percy & Kastrukoff 2001; J. R. Percy & E. Redelmeier 2003, in preparation). Alternatively, the residuals may be due to additional lower amplitude modes or other sources of irregular variability such as convection. If there are large convection cells in the outer layers of the stars, then they may produce brightness variations on the convective turnover time. They may also produce rotational variations if different hemispheres of the stars have different average brightnesses. They may also produce deviations from radial symmetry that will perturb the radial modes. Residuals could also occur if the oscillations were nonsinusoidal. We looked for harmonic peaks in the power spectra, but there were none above the noise level. Also, the monoperiodic variables shown in Figures 1–7 of PWH have light curves that appear to be sinusoidal.

Lebzelter (1999) and Lebzelter, Kiss, & Hinkle (2000) have raised the same issue in connection with radial velocity observations of semiregular variables, but recent measurements (Lebzelter & Hinkle 2002) show that while there may be some deviation from regular light variability due to convective cells, the main cause is most likely stellar pulsation.

Another caveat is that our results depend slightly on whether we fit each season of data with the mean periods given above or with the periods that were actually determined for that season. We have assumed that each star has permanent radial pulsation periods and that the mean periods are the best approximation to these.

The amplitudes of the dominant radial modes can change from season to season, and they have a tendency to rise and fall on a timescale of 2500–3500 days. This timescale is much longer than the "long secondary periods" of brightness variation that occur in some of these stars. RZ Ari, V523 Mon, and BC CMi do not have known long secondary periods; the long secondary periods of UX Lyn and FS Com are 420 and 675 days, respectively. The growth rates η (the fractional increase in amplitude per cycle) for models appropriate for our five SAPRGs are in the range 0.03–0.15 (Fox & Wood 1982; Ostlie & Cox 1986), so the timescales are consistent with the natural growth (or decay) times for the modes.

As usual, our results would be firmer if we had longer data set on more stars. It would also help to have theoretical models that are more specifically designed for these stars, rather than for Mira stars.

5. CONCLUSIONS

At least five small-amplitude pulsating red giants show strong evidence of multiperiodicity. The periods, and their ratios, are consistent with low-order radial pulsation. The amplitudes of the dominant modes vary on timescales of 2500–3500 days. In a given star, there is some tendency for the amplitudes of the two or three modes to rise and fall in unison.

REFERENCES

Dumm, T., & Schild, H. 1998, NewA, 3, 137
Eyer, L., & Grenon, M. 1997, in *Hipparcos* Venice '97, ed. B. Battrick (ESA SP-402; Noordwijk: ESA), 467
Foster, G. 1995, AJ, 109, 1889
Fox, M. W., & Wood, P. R. 1982, ApJ, 259, 198
Grenon, M. 1993, in ASP Conf. Ser. 40, Inside the Stars, ed. W. Weiss & A. Baglin (San Francisco: ASP), 693
Henry, G. W., Fekel, F. C., Henry, S. M., & Hall, D. S. 2000, ApJS, 130, 201
Houk, N. 1963, AJ, 68, 253
Jorgensen, H. E., & Petersen, J. O. 1967, Z. Astrophys., 67, 377
Jorissen, A., Mowlavi, N., Sterken, C., & Manfroid, J. 1997, A&A, 324, 578
Kiss, L. L., Szatmáry, K., Cadmus, R. R., & Mattei, J. A. 1999, A&A, 346, 542
Koen, C., & Laney, C. 2000, MNRAS, 311, 636
Lebzelter, T. 1999, A&A, 351, 644

Lebzelter, T., & Hinkle, K. H. 2002, A&A, 393, 563
Lebzelter, T., Kiss, L. L., & Hinkle, K. H. 2000, A&A, 361, 167
Ostlie, D. A., & Cox, A. N. 1986, ApJ, 311, 864
Percy, J. R., & Bakos, A. G. 2003, in The Garrison Festschrift, ed. R. O. Gray, C. J. Corbally, & A. G. D. Philip (Schenectady: L. Davis Press), in press
Percy, J. R., & Hosick, J. 2002, MNRAS, 334, 669
Percy, J. R., & Kastrukoff, R. 2001, J. AAVSO, 30, 16
Percy, J. R., & Parkes, M. 1998, PASP, 110, 1431
Percy, J. R., Wilson, J. B., & Henry, G. W. 2001, PASP, 113, 983
Sperl, M. 1998, Comm. Asteroseismology (Univ. Vienna), 112
Tufte, E. R. 1990, Envisioning Information (Cheshire: Graphics Press), 46
Tukey, J. 1972, in Statistical Papers in Honor of George W. Snedecor, ed. T. A. Bancroft (Ames: Iowa State Univ.), 296
Wood, P. R. 2000, Publ. Astron. Soc. Australia, 17, 8
Xiong, D. R., Deng, L., & Cheng, Q. L. 1998, ApJ, 499, 355